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Department of Computer Science and Engineering University of Minnesota 4-192 Keller Hall 200 Union Street SE Minneapolis, MN 55455-0159 USA

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TMA: Trajectory-based Multi-Anycast for Multicast Data Delivery in Vehicular Networks

Jaehoon Jeong, Tian He, and David Du

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Jaehoon Jeong, Tian He and David H.C. Du
Department of Computer Science & Engineering, University of Minnesota
Email: {jjeong,tianhe,du}@cs.umn.edu

Abstract—This paper describes Trajectory-based Multi-Anycast (TMA) Forwarding, tailored and optimized for the multicast data delivery in vehicular networks. To our knowledge, this is the first attempt to investigate the efficient multicast data delivery in vehicle networks based on the trajectories of vehicles in the multicast group. Due to the privacy concern, we assume only a central server knows the trajectory of each vehicle and the estimated current location of the vehicle. Therefore, after receiving a request of multicast data delivery from a source vehicle, the central server has to figure out how the data has to be delivered to the vehicles in the multicast group. For a given target vehicle in the multicast group, multiple packet-andvehicle rendezvous points are computed as a set of relay nodes to temporarily hold the data, considering the vehicle's trajectory. This set of rendezvous points can be considered an Anycast set for the target vehicle. We have formulated the multicast data delivery as the data delivery to the anycast sets of the multicast group vehicles. Through theoretical analysis and extensive simulation, it is shown that our design provides an efficient multicast for moving vehicles under a variety of vehicular traffic conditions.

I. INTRODUCTION

Vehicular Ad Hoc Networks (VANET) to support the Intelligent Transportation Systems (ITS) have recently become one of promising wireless networking research areas [1]–[6]. This trend is due to Dedicated Short Range Communications (DSRC) standardization [7] and the GPS-based navigation at an unprecedented rate [8]. It can be easily envisioned that the vehicular networking will be realized by integrating the cutting-edge DSRC and GPS technologies. Therefore, with this trend, we can raise one research question of how to take advantage of these GPS-guided driving paths (called *vehicle trajectories*) in order to achieve better data forwarding performance in vehicular networks.

Our goal in this paper is to design an efficient multicast scheme to provide drivers and passengers with the *customized services through vehicular networks* based on the *vehicle trajectories*. Such services include message dispatch (e.g., taxi calling and police-car calling), data pulling services (e.g., stock market and news), location-based services (e.g., cheap gas stations and popular restaurants), and multi-media data sharing (e.g., popular songs and video clips).

The current multicast approaches [9], [10] for vehicular networks are not fully addressing the following properties of road networks: (i) Road network layout, (ii) Vehicle mobility along the roadways with a finite speed, and (iii) Vehicle trajectory guided by GPS navigator. These three properties in road networks give us an opportunity to design an efficient

multicast data delivery in vehicular networks in terms of reducing delivery cost (e.g., channel utilization). Our paper proposes <u>Trajectory-based Multi-Anycast</u> (TMA) considering these properties of vehicular networks. To the best of our knowledge, our TMA is the first attempt to investigate the vehicle trajectory for the efficient multicast data delivery in a privacy-preserving manner.

The first challenge is how to select packet-and-vehicle rendezvous points for multicasting. With the vehicle travel delay and the packet delivery delay distributions, our TMA algorithm determines multiple rendezvous points (a set of relay nodes to temporarily hold data packets) of the destination vehicle and the packet. These rendezvous points are called *target points* in this paper and can be considered as an Anycast set for the destination vehicle. Thus, we formulate the multicast data delivery to multiple destination vehicles in the multicast group as to deliver data to any target points in the anycast sets of those destination vehicles. This multicast approach to multiple anycast sets is called *Trajectory-based Multi-Anycast*.

The second challenge is how efficiently to connect these anycast sets by selecting one target point per anycast set (called representative target point) into a multicast tree, guaranteeing a given data delivery ratio. Our TMA algorithm constructs a Delivery-Ratio Constrained Minimum Steiner Tree with the representative target points of the anycast sets to minimize the total channel utilization [11]. Note that our multicast tree consists of relay nodes that are either relay nodes or destination nodes and its root node is AP to disseminate the data packet, and also that this multicast tree is updated by the central server (knowing the vehicle trajectories) while the multicast group vehicles are moving in the road networks. Our intellectual contributions are as follows:

- The multicast data delivery architecture in vehicular networks,
- The modeling for packet delivery delay, vehicle travel delay, and link cost, and
- An optimal target point selection algorithm for multiple destination vehicles.

The rest of this paper is organized as follows: Section II describes the problem formulation. Section III explains the packet and vehicle delay models. Section IV explains our TMA design. Section V explains our TMA protocol. Section VI evaluates our design. We summarize related work in Section VII and conclude this paper in Section VIII.

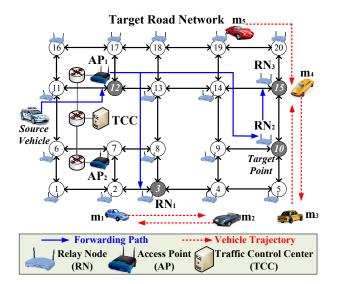


Fig. 1. Multicast Forwarding in Vehicular Network

II. PROBLEM FORMULATION

In this section, we formulate the multicasting in vehicular networks as follows: Given a road network with APs, our goal is to deliver packets reliably from source vehicle (or AP) to multicast group vehicles at the required End-to-End data delivery ratio, while minimizing the delivery cost (e.g., channel utilization and delay) in vehicular networks.

A. Assumptions

The following set of assumptions is used:

- Access Point (AP) is a wireless network node connected to the wired network (e.g., the Internet) with DSRC device, storage and processor in order to provide vehicles with the wired network connectivity. For the cost effectiveness, as shown in Fig. 1, APs are sparsely deployed into road networks and are interconnected with each other through the wired network or wirelessly (as Mesh Network) for the data forwarding [1], [12]. Each AP installation with power and wired network connectivity can cost as high as US\$5,000 [13]. The geographical location information of APs is available to vehicles.
- Traffic Control Center (TCC) is a trustable entity that maintains vehicle trajectories without exposing the vehicle trajectories to other vehicles for privacy concerns. We can integrate vehicular networks to the existing TCCs used for the road traffic engineering [14]. As shown in Fig. 1, TCC and APs are interconnected with each other through the wired network, such as the Internet.
- Relay Node (RN) is a temporary packet holder with DSRC device, storage, and processor that is a stand-alone node without the wired network connectivity to APs, as shown in Fig. 1. With a small number of APs, RNs can support the reliable data delivery at a low cost. For the sake of clarity, RN is assumed to be deployed at each intersection. This assumption is relaxed in Section V-B.
- Vehicles participating in VANET have DSRC device [7].
 Nowadays many vehicle vendors, such as GM and Toy-

- ota, are planning to release vehicles with DSRC devices [15], [16].
- Vehicles, APs and RNs are installed with GPS-based navigation systems and digital road maps [17], [18]. Traffic statistics, such as vehicle arrival rate λ and average vehicle speed v per road segment, is available via commercial navigation systems (e.g., Garmin [17]).
- Drivers input their travel destination into their GPS-based navigation systems before their travel and so their vehicles can compute their future trajectory based on their current location and their final destination. Multicast-service-participatory vehicles regularly report their trajectory information and their current location to TCC through APs.

B. Relay-Node-Assisted Forwarding

In this subsection, we justify our vehicular network architecture containing relay nodes. In order to support the *justin-time* data delivery from AP to destination vehicles, the delivery delay variation in the packet forwarding path should be bounded. Otherwise, the packets will miss the destination vehicles because they may arrive at the target points later than the destination vehicles.

Without relay nodes, the data forwarding schemes based on *stochastic model* (e.g., VADD [5]) cannot be used to deliver from AP to mobile destination vehicles. Note that in the stochastic model, each vehicle tries to forward its packets opportunistically to a better neighboring vehicle toward the packet destination, so this packet delivery process is a random walk. However, this stochastic-model-based forwarding has a huge delay variation, so it cannot be used for the multi-hop infrastructure-to-vehicle data delivery, as shown in [19].

To reduce the delivery delay variation, we deploy relay nodes as packet store-and-forward nodes. In our model, the packet is source-routed via relay nodes at intersections. Our model has a more accurate packet delay model than the stochastic model, so the *just-in-time* delivery can be realized from AP to destination vehicles.

C. The Concept of Multi-Anycast

In this paper, we define a new concept of *Multi-Anycast* as follows:

Definition 2.1 (Multi-Anycast): Multi-Anycast is the multicast to anycast sets where an anycast set is a set of target points among a multicast group vehicle's future intersections on its vehicle trajectory.

We will explain the concept of Multi-Anycast using Fig. 1. In this figure, *Source Vehicle* sends its data packet to AP (denoted as AP_1) in relay-node-assisted unicast [20] where the target point is AP. The AP will multicast the packet to the multicast group vehicles, as shown in Fig. 1. Table I shows the trajectories of the multicast group vehicles (m_i for i=1..5) and the corresponding anycast set A_i for m_i in the figure.

AP can send a packet to any target point in anycast set A_i . As a result, the forwarding toward any target point in the anycast set leads to *Anycast*. For example, in Table I, vehicle

TABLE I
MULTI-ANYCAST FOR MULTICAST GROUP VEHICLES

Vehicle	Vehicle Trajectory	Anycast Set A_i	Target
m_1	$n_2 ightarrow n_3 ightarrow n_4 ightarrow n_5$	$\{n_3, n_4, n_5\}$	n_3
m_2	$n_4 \rightarrow n_3 \rightarrow n_2 \rightarrow n_1$	$\{n_3, n_2, n_1\}$	n_3
m_3	$n_5 \rightarrow n_{10} \rightarrow n_{15} \rightarrow n_{20}$	$\{n_{10}, n_{15}, n_{20}\}$	n_{10}
m_4	$n_{15} \rightarrow n_{10} \rightarrow n_5 \rightarrow n_4$	$\{n_{10}, n_5, n_4\}$	n_{10}
m_5	$n_{19} \rightarrow n_{20} \rightarrow n_{15}$	$\{n_{20}, n_{15}\}$	n_{15}

 m_1 has its trajectory of $n_2 \rightarrow n_3 \rightarrow n_4 \rightarrow n_5$. For vehicle m_1 , the anycast set is $\{n_3, n_4, n_5\}$ for the delivery ratio α . In this anycast set, any target point can be selected as a packet destination by AP. In this figure, vehicle m_1 selects n_3 as a target point.

Our Multi-Anycast problem can be defined as follows: How to multicast packets to the anycast sets, specifically, to an optimal target point for each anycast set related to a multicast group vehicle in order to minimize the overall multicast delivery cost?

To answer this problem, in Section III, we first model the packet delivery delay and the vehicle travel delay used for the estimation of *just-in-time* delivery. Next, in Section IV, with the probability distributions of the packet delivery delay and the vehicle travel delay, we will explain the target point selection and the construction of a multicast tree.

III. DELAY MODELS

In this section, we describe three types of delay models for the *just-in-time* delivery, proposed in our TSF scheme [19]: (i) Link delay model, (ii) Packet delay model, and (iii) Vehicle delay model.

A. Link Delay Model

This subsection analyzes the link delay for one road segment with one-way vehicular traffic given the road length (l), the vehicle arrival rate (λ) , the vehicle speed (v) and the communication range (R). It is supposed that for packet storeand-forward, one relay node is placed at each end-point (i.e., intersection) of the road segment. Note that the link delay for a two-way road segment is left as future work.

It is notable that in the VANET scenarios, the carry delay is dominant delay factor because it is *several orders-of-magnitude* longer than the communication delay. Thus, in our analytical model for the link delay, the carry delay is focused for the sake of clarity, although the small communication delay does exist in our design.

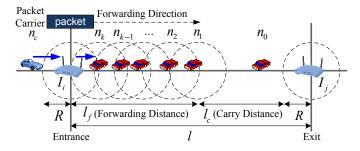


Fig. 2. Link Delay Modeling for Road Segment

The link delay for one road segment is computed considering the following two cases for the communication range of the relay node at intersection I_i in Fig. 2:

- Case 1: Immediate Forward: When the current packet carrier n_c arrives at the entrance intersection I_i , there is at least one vehicle (i.e., k > 0) moving toward the intended next intersection along the packet's forwarding path. In this case, n_c forwards its packets to the ad-hoc network consisting of vehicles moving toward the exit intersection I_i of the road segment.
- Case 2: Wait and Carry: When the current packet carrier n_c arrives at the entrance intersection I_i , there is no vehicle (i.e., k=0) moving toward the intended next intersection along the packet's forwarding path. In this case, n_c forwards its packets to the relay node at the entrance intersection I_i . The relay node holds the packets until another vehicle is moving toward the exit entrance I_j .

Thus, the expectation and variance of the link delay can be computed with the link delays of these two cases as follows:

$$d = \begin{cases} \frac{l - l_f - R}{v} & \text{for case 1: immediate forward,} \\ \frac{1}{\lambda} + \frac{l - R}{v} & \text{for case 2: wait and carry.} \end{cases}$$
 (1)

$$E[d] = E[\text{link delay} \mid \text{forward}] \times P[\text{forward}] + E[\text{link delay} \mid \text{wait}] \times P[\text{wait}].$$
 (2)

$$Var[d] = E[d^2] - (E[d])^2$$
 (3)

Refer to Appendix B and Appendix C for the detailed derivation of (2) and (3), respectively.

Let G=(V,E) be road network graph where V is the set of intersections and E is the matrix of road segments. With the mean E[d] and variance Var[d] of the link delay, we model the link delay d as the Gamma distribution. Note that the Gamma distribution is usually used to model the positive continuous random variable, such as the waiting time and lifetime [21]. Thus, the distribution of the link delay d_i for the edge $e_i \in E[G]$ is $d_i \sim \Gamma(\kappa_i, \theta_i)$ such that $E[d_i] = \kappa_i \theta_i$ and $Var[d_i] = \kappa_i \theta_i^2$ for $d_i, \kappa_i, \theta_i > 0$ [21]. Since we have the mean and variance of the link delay, that is, $E[d_i] = \mu_i$ in (2) and $Var[d_i] = \sigma_i^2$ in (3), we can compute the parameters θ_i and κ_i of the Gamma distribution [21].

Note that our design can accommodate an empirical link delay distribution if available through measurement. For this empirical distribution of link delay, adjacent relay nodes can periodically exchange probe packets with each other to obtain link delay samples. Therefore, with the link delay model for a directed edge corresponding to a road segment, we will be able to model the End-to-End packet delay and the vehicle travel delay in the next subsections.

B. E2E Packet Delay Model

In this subsection, we model the End-to-End Packet Delay from one position to another position in a given road network. As discussed in Section III-A, the link delay is modeled as the Gamma distribution of $d_i \sim \Gamma(\kappa_i, \theta_i)$ for edge $e_i \in E(G)$ in

the road network graph G. Given a forwarding path from AP to a target point, we assume that the link delays of edges consisting of the path are independent. From this assumption, the mean and variance of the E2E packet delay are computed as the sum of the means (E[P]) and the sum of the variances (Var[P]) of the link delays along the E2E path, respectively. Therefore, the E2E packet delay distribution can be modeled as $P \sim \Gamma(\kappa_p, \theta_p)$ such that $E[P] = \kappa_p \theta_p$ and $Var[P] = \kappa_p \theta_p^2$ for $P, \kappa_p, \theta_p > 0$ [21].

C. Vehicle Delay Model

In this subsection, we model the Vehicle Delay from one position to another position in a given road network in the same way as the Packet Delay Model in Section III-B. Given the road network graph G, the travel time for edge $e_i \in E(G)$ is modeled as the Gamma distribution of $t_i \sim \Gamma(\kappa_i, \theta_i)$; note that the travel time distribution for each road segment can be obtained through vehicular traffic measurement and it is usually considered the Gamma distribution by the civil engineering community [22], [23]. The parameters κ_i and θ_i of the Gamma distribution are computed with the mean travel time μ_i and the travel time variance σ_i^2 using the relationship among the mean $E[t_i]$, the variance $Var[t_i]$, κ_i , and θ_i such that $E[t_i] = \kappa_i \theta_i$ and $Var[t_i] = \kappa_i \theta_i^2$ for $t_i, \kappa_i, \theta_i > 0$ [21] in the same way with E2E Packet Delay Model in Section III-B.

Given a vehicle trajectory from the vehicle's current position to a target point, we suppose that the travel times of edges consisting of the trajectory are independent. From this assumption, the mean and variance of the vehicle travel delay are computed as the sum of the means (E[V]) and the sum of the variances (Var[V]) of the edge travel delays along the trajectory, respectively. Therefore, the E2E vehicle delay distribution can be modeled as $V \sim \Gamma(\kappa_v, \theta_v)$ such that $E[V] = \kappa_v \theta_v$ and $Var[V] = \kappa_v \theta_v^2$ for $V, \kappa_v, \theta_v > 0$ [21].

So far, we have explained our delay models. In the next section, based on these delay models, we will explain our Multi-Anycast design in detail.

IV. MULTI-ANYCAST DESIGN

In this section, we explain how to perform Multi-Anycast for multicast group vehicles. We can formulate the optimization of Multi-Anycast data delivery as follows:

Let G=(V,E) be a road network graph where V is the set of intersections n_i and E is the matrix of road segments e_{ij} whose values are the pairs of physical distance l_{ij} and packet link delay d_{ij} between n_i and n_j . Let M be a set of multicast group vehicles m_i such that $m_i \in M$. Let $V_i = V(n_i)$ be the vehicle travel delay of vehicle m_i from its current position to its target point n_i . Let $P_i = P(n_i)$ be the packet delivery delay from packet source (i.e., AP) to the target point n_i . Let A_i be the set of target points (called $anycast\ set$) for vehicle m_i such that (i) $A_i = \{a_{i1}, a_{i2}, ..., a_{is_i}\}$ for $s_i = |A_i|$ and (ii) $Pr[P(a_{ij}) \leq V(a_{ij})] \geq \alpha$ for $j = 1..s_i$. Let a_i^* be an optimal target point in A_i such that the cost from AP to the target point a_i^* is minimum. Let T be a multicast tree for multicast group M. Let Cost(T) be a multicast delivery cost for the

tree T; that is, the sum of edge weights such that the edge weight is the link channel utilization (defined as the number of transmissions) in the edge, formally defined in Section IV-B.

Our goal is to construct a minimum-cost multicast tree from the packet source (AP) to all multicast group vehicles while guaranteeing a given data delivery ratio α . The following optimization finds an optimal multicast tree T^* to satisfy our goal:

$$T^* \leftarrow \arg\min_{T \subseteq G} E[Cost(T)] \tag{4}$$

subject to $Pr[P_i \leq V_i] \geq \alpha$ for $n_i \in V[T]$ where n_i is a target point of vehicle m_i . In (4), an optimal multicast tree T^* is the Delivery-Ratio Constrained Minimum Steiner Tree from packet source AP to the target points n_i for all of the multicast group vehicles [11]. Therefore, for a given multicast group M, Multi-Anycast can be formally defined as follows: Multi-Anycast is the packet forwarding scheme from packet source AP to multicast group M with the minimum mulicast delivery cost such that for each anycast set A_i per multicast group vehicle $m_i \in M$, AP multicasts its packet to one target point a_{ij} in the anycast set A_i .

We explain this optimization for the *Multi-Anycast* into the following three steps: First, we explains how to compute an anycast set of target points A_i per multicast group vehicle m_i . Second, we describe how to select an optimal target point a_i^* per anycast set A_i . Last, we explain how to construct a minimum-cost multicast tree with the selected target points such that the multicast tree satisfies the required data delivery ratio α .

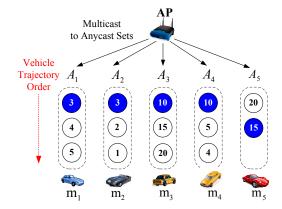


Fig. 3. Anycast Sets consisting of Target Points

A. Step 1: Constructing Anycast Sets

In this subsection, we explain how to construct an anycast set of target points A_i per multicast group member $m_i \in M$ with the packet delay distribution and vehicle delay distribution. The target point selection is based on the *delivery probability* that the packet will arrive at the target point earlier than the destination vehicle. This delivery probability can be computed with the packet's delivery delay distribution and the destination vehicle's travel delay distribution.

We now explain how to construct any cast set A_i with the trajectory of each multicast group vehicle m_i . Given a data delivery ratio α , we select target points a_{ij} from the intersections

on m_i 's trajectory such that $Pr[P(a_{ij}) \leq V(a_{ij})] \geq \alpha$; note that $Pr[P(a_{ij}) \leq V(a_{ij})]$ is the probability that the packet sent by AP will arrive earlier at target point a_{ij} than the destination vehicle m_i . Thus, for each vehicle m_i , we can compute the anycast set A_i , while guaranteeing the required data delivery ratio α as follows:

$$A_i = \{a_{ij} | Pr[P(a_{ij}) \le V(a_{ij})] \ge \alpha \text{ for } j \in I_i\}$$
 (5)

where I_i is a set of intersections on the trajectory of vehicle m_i . For example, as shown in Fig. 3, we assume that there are five vehicles in a multicast group, denoted as m_i for i=1..5. By (5), we can compute an anycast set for each vehicle m_i , as shown in Fig. 3.

Now we explain how to compute the delivery probability. As a reminder, the packet delay distribution and the vehicle delay distribution can be computed as explained in Section III-B and Section III-C, respectively. The probability distributions of the packet delay P and the vehicle delay V are assumed to be the Gamma distributions such that $P \sim \Gamma(\kappa_p, \theta_p)$ and $V \sim \Gamma(\kappa_v, \theta_v)$ [21]. Assuming that the packet delay distribution and the vehicle delay distribution are independent of each other, the delivery probability $Pr[P_i \leq V_i]$ for target point n_i is computed as follows:

$$Pr[P_i \le V_i] = \int_0^{TTL} \int_0^v f(p)g(v)dpdv. \tag{6}$$

where f(p) is the probability density function (PDF) of packet delay $p,\ g(v)$ is the PDF of vehicle delay v, and TTL is the packet's Time-To-Live (TTL). Note that the delivery probability is computed considering the packet's lifetime TTL; that is, since the packet is discarded after TTL, the probability portion is zero after TTL.

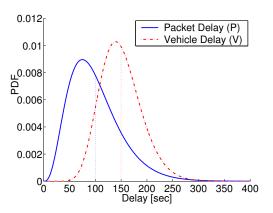


Fig. 4. Packet Delay Distribution and Vehicle Delay Distribution

For example, Fig. 4 shows the distribution of packet delay P from access point AP_1 to target point n_{12} (along the forwarding path shown in Fig. 1) and the distribution of vehicle delay V from vehicle m_1 's current position n_2 to target point n_{12} (along vehicle m_1 's trajectory shown in Fig. 1).

Note that by the delivery probability in (6), the target point selection depends on the packet delay model P and the vehicle delay model V that are described in Section III.

However, our packet delay model and vehicle delay model are not restricted to the Poisson vehicle arrival model and the Gamma distribution model, respectively. Our TMA can accommodate any empirical distributions. That is, if more accurate distributions are available, our TMA can use them for the computation of the delivery probability.

B. Step 2: Selecting Target Point Points from Anycast Sets

In this subsection, we explain how to select an optimal target point a_i^* as a representative target point for each anycast set A_i per multicast group member $m_i \in M$. We select a target point a_i^* that corresponds to the shortest-path endpoint from source node AP to anycast set A_i in terms of the path cost $Cost(a_{ij})$, which is the sum of edge weights (e.g., link channel utilization values) as follows:

$$a_i^* \leftarrow \arg\min_{a_{ij} \in A_i} Cost(a_{ij}).$$
 (7)

For example, for five anycast sets in Fig. 3, the representative target points (denoted as blue-color nodes) are selected such that they are the shortest-path endpoints from AP to A_i for i=1..5. These selected anycast representative target points become packet destination nodes for the multicast tree in the next step, discussed in Section IV-C.

Note that our target point selection algorithm from anycast sets cannot make an optimal set of target points for the overall multicast tree cost. The selection of one target point per anycast set as a destination node in an optimal multicast tree is itself NP-Complete problem. However, our selection algorithm can make an optimal shortest path tree used as the initial multicast tree in the next step, explained in Section IV-C.

As mentioned before, we define the *link channel utilization* as the number of transmissions in a road segment used for the packet forwarding over the road segment as follows. Fig. 2 shows the forwarding distance l_f of the vehicular ad-hoc network that consists of vehicles connected by the communication range R. This ad-hoc network has the forwarding distance corresponding to the extended communication range of k vehicles, that is, l_f . Thus, the link cost w_{ij} for road segment (I_i, I_j) is the number of transmissions on the road segment (from the relay node at I_i to the header vehicle n_1 of the ad-hoc network) such that $w_{ij} = \lceil E[l_f]/R \rceil$. For $E[l_f]$, we derive the average forwarding distance $E[l_f]$ for the road segment (I_i, I_j) in Appendix A.

C. Step 3: Constructing Multicast Tree for Multi-Anycast

In this subsection, we explain how to build a Delivery-Ratio Constrained Minimum Steiner Tree for multicasting data packets to anycast sets where the constraint is the data delivery ratio α . In the previous step, we select the anycast representative target points as packet destination nodes of the multicast tree. It is known that constructing a Constrained Minimum Steiner Tree itself is NP-Complete problem. To construct our Constrained Minimum Steiner Tree, we use the Bounded Shortest Multicast Algorithm (BSMA) proposed in [11]. Note that for BSMA, our constraint for multicast tree is the *data delivery ratio* rather than the *data delivery*

delay. Since our TMA design is independent of multicast tree algorithm, any efficient multicast tree algorithm can be used for the Delivery-Ratio Constrained Minimum Steiner Tree.

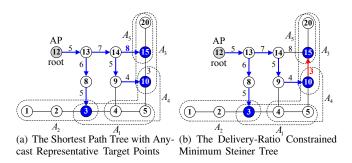


Fig. 5. TMA Multicast Tree Construction

As an input for this algorithm, we construct the shortest path tree by merging the shortest paths from AP to the representative target points a_i^* of any cast sets A_i . Fig. 5(a) shows the shortest path tree made from five any cast sets A_i for i = 1..5. With the initial multicast tree as shown in the figure, the BSMA algorithm searches better sub-paths between an arbitrary pair of two multicast nodes (e.g., relay node or destination node) in order to enhance the multicast tree in terms of multicast delivery cost, while satisfying the data delivery ratio α to each target point. Fig. 5(b) shows a better multicast tree by replacing the path $n_{14} \rightarrow n_{15}$ with the path $n_{14} \rightarrow n_9 \rightarrow n_{10} \rightarrow n_{15}$ to reduce the overall tree cost. Refer to [11] for the detailed algorithm of BSMA. Therefore, we can construct a Delivery-Ratio Constrained Minimum Steiner Tree to perform Multi-Anycast to the multicast group. In next section, we will explain Multi-Anycast Protocol to deliver data packets to multicast group vehicles using a multicast tree discussed in this section.

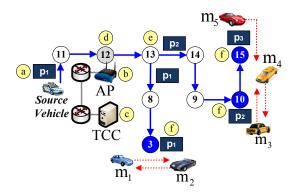


Fig. 6. TMA Multi-Anycast Protocol
V. MULTI-ANYCAST PROTOCOL

In this section, we explain our Multi-Anycast forwarding procedure and optimization issues in the multicast forwarding.

A. Forwarding Procedure

Our Multi-Anycast protocol supports vehicle-to-multicast-group data delivery in the following six steps, as shown in Fig. 6. (a) Source vehicle sends a data packet toward AP in unicast. (b) AP forwards the packet to TCC through the wired

network. (c) TCC computes a multicast tree for the multicast group vehicles. (d) TCC sends AP the packet with the multicast tree encoded. (e) The packet is source-routed from AP to the destination nodes of the multicast tree. (f) The packet copies arrive at the relay nodes corresponding to the destination nodes and will wait for the multicast group vehicles.

B. TMA Optimization Issues

We consider the following optimization issues for the practical deployment of TMA systems: (i) TMA forwarding with multiple APs in large-scale road networks, (ii) The scalable TMA systems with multiple servers and TCCs, and (iii) The partial deployment of relay nodes.

First, with multiple APs, we can support our TMA protocol in a large-scale road network. For each multicast group vehicle, we find an AP among the APs whose delivery cost to the vehicle is minimum and compute the shortest-cost path from the AP to the vehicle. We construct one multicast tree for each AP with the shortest-cost paths originating from the AP. We apply our multicast tree optimization algorithm called BSMA (discussed in Section IV-C) to each multicast tree. With these optimized multicast trees, we can perform multicasting by letting each multicast tree disseminating the packet copies to the multicast group vehicles belonging to the tree at the moment.

Second, in a large-scale road network, one Traffic Control Center (TCC) might not scale up to provide a large number of vehicles with the TMA multicast. TCC can have multiple servers having the replicas of the trajectories and also the large-scale road network can be divided into multiple regions that have their own TCC for the TMA multicast. Each TCC per region performs the TMA multicast in the centralized way with the trajectory information.

Third, the partial deployment of relay nodes allows that some intersections might not have their own relay nodes. In this case, we filter out the edges without Relay Node (RN) from the road network graph. With this filtered graph, we can run our target point selection algorithm in Section IV-A without any change. Clearly, as the number of relay nodes decreases, the data delivery probability from the AP to the destination vehicle will decrease. Also, it is important to investigate how to deploy a certain number of relay nodes in order to guarantee the required delivery delay and delivery ratio. This deployment issue is left as future work.

So far, we have explained our Multi-Anycast protocol for the multicast data delivery from source vehicle to multicast group vehicles via TCC, AP and RNs. Next, we will show the performance of our TMA in a variety of vehicular network setting.

VI. PERFORMANCE EVALUATION

In this section, we evaluate the performance of *TMA*, performing on an optimal target point selection from anycast sets for the multicast tree construction, described in Section IV. The evaluation setting is as follows:

- **Performance Metrics:** We use (i) *Delivery cost*, (ii) *Delivery delay*, and (iii) *Delivery ratio* as metrics.
- **Baselines:** Our work is the first attempt for the multicast data forwarding based on the vehicle trajectory, so we have no other state-of-the-art schemes for comparison. To evaluate *TMA*, we compare it with the following two baselines: (i) Random Target Point Selection (*Random*) and (ii) Network-Wide Packet Flooding (*Flood*). In *Random*, a target point is randomly selected from each vehicle's trajectory. We construct the shortest path tree from AP to the selected target points in terms of delivery delay. In *Flood*, the packet is copied at intersections into as many packet copies as the outgoing intersections. Duplicate packets are discarded by relay nodes holding the same packet.
- **Parameters:** In the performance evaluation, we investigate the impacts of (i) *Vehicular traffic density* N, (ii) *Vehicle speed* μ_v , and (iii) *Vehicle speed deviation* σ_v .

TABLE II
SIMULATION CONFIGURATION

Parameter	Description	
	The number of intersections is 49.	
Road network	The area of the road map is 8.25km×9km	
	(i.e., 5.1263miles×5.5923miles).	
Communication range	R = 200 meters (i.e., 656 feet).	
Number of vehicles	The number N of vehicles moving within	
(N)	the road network. The default of N is 160.	
	The expiration time of a packet. The	
Time-To-Live	default TTL is the vehicle trajectory's	
(TTL)	lifetime, that is, the vehicle's travel time	
	for the trajectory, i.e., 2, 100 seconds.	
	$v \sim N(\mu_v, \sigma_v)$ where $\mu_v = \{20, 25,,$	
Vehicle speed	60} MPH and $\sigma_v = \{1, 2,, 10\}$ MPH.	
(v)	The maximum and minimum speeds are	
	$\mu_v + 3\sigma_v$ and $\mu_v - 3\sigma_v$, respectively.	
	The default of (μ_v, σ_v) is $(40, 5)$ MPH.	
	Let $d_{u,v}$ be the shortest path distance	
Vehicle travel	from start position u to end position v in	
path length	the road network. $l \sim N(\mu_l, \sigma_l)$ where	
(l)	$\mu_l = d_{u,v}$ km and $\sigma_l = 3$ km (1.86miles).	

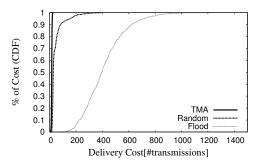


Fig. 7. CDF of Delivery Cost

We have built a simulator based on the scheduler provided by SMPL [24] in C with the following settings. A road network with 49 intersections is used in the simulation setting described in Table II. One Access Point (AP) is deployed in the center of the network and is connected to Traffic Control Center (TCC). Each vehicle's movement pattern is determined by a *Hybrid Mobility model* of City Section Mobility model [25] and Manhattan Mobility model [26] suitable for vehicle mobility in urban areas. Note that for any road network topology, our TMA can accommodate *any vehicle mobility* because it can accommodate the empirical distributions of packet delivery delay and vehicle travel delay, as discussed in Sections III and IV.

During the simulation, following an exponential distribution with a mean of 5 seconds, packets are dynamically generated from AP in the road network. The total number of generated packets is 1000 and the simulation is continued until all of these packets are either delivered or dropped due to TTL expiration. The system parameters are selected based on a typical DSRC scenario [7]. Unless otherwise specified, the default values in Table II are used.

A. Forwarding Behavior Comparison

We compare the forwarding behaviors of TMA, Random and Flood with the cumulative distribution function (CDF) of packet delivery cost; note that for TMA, the data delivery ratio threshold α is 95%. From Fig. 7, it is very clear that TMA has lower delivery cost than Random, and also much lower delivery cost than Flood. That is, for any given CDF value from the vertical axis in the figure, TMA always has lower cost in the horizontal axis than Random and Flood. For example, TMA needs a delivery cost of 18 transmissions for 86% CDF while for the same CDF value, Random needs 68 transmissions and Flood needs 584 transmissions. In other words, on the delivery cost, TMA requires 26% transmission of Random and 0.03% transmission of Flood, respectively. We will show the forwarding performance of these three schemes quantitatively in the following subsections.

B. Impact of Vehicle Number

The number of vehicles in the road network determines the vehicular traffic density in a road network. In this subsection, we intend to study how effectively TMA can forward packets from AP towards the multicast group vehicles using their vehicle trajectories. Through our extensive simulations, we observe that under any vehicular traffic density, TMA significantly outperforms Random and Flood in terms of the average delivery cost per packet to the multicast group. Fig. 8(a) shows the packet delivery cost comparison among TMA, Random and Flood with varying the number of vehicles, that is, from 60 to 240. From this figure, TMA has lower packet delivery cost than Random and Flood at all vehicular densities. The observed trend is that the delivery cost in TMA is almost stable even though the number of vehicles increases. This is because TMA always tries to construct a minimum-cost multicast tree. On the other hand, Flood needs higher delivery cost as the vehicular density increases. This is because the higher vehicular density generates more duplicate packets in flooding, leading to the more transmissions. Random has a constant stable curve similar to TMA's curve due to its randomness in target point selection. For the average transmission number,

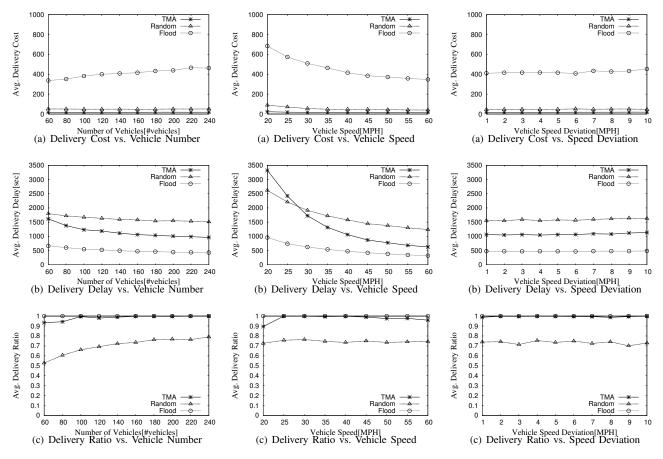


Fig. 8. Impact of Vehicle Number N

Fig. 9. Impact of Vehicle Speed μ_v

Fig. 10. Impact of Vehicle Speed Deviation σ_v

as shown in Fig. 8(a), *Flood* has 23 times more transmissions than *TMA*. *Random* has almost 2.8 times more transmissions than *TMA*.

For the delivery delay, as shown in Fig. 8(b), as the vehicular density increases, the delivery delay decreases and the delivery ratio increases. This is because the more vehicles increase the forwarding probability among vehicles, so this reduces the carry delay, leading to the overall shorter delivery delay. *TMA* has 2.3 times longer delay than *Flood*, but has only 72% delay of *Random*. From Fig. 8(a) and Fig. 8(b), *TMA* takes 2.3 times longer delivery delay of *Flood*, but needs only 4.3% delivery cost of *Flood*. Thus, even though *TMA* sacrifices the delivery delay compared with *Flood*, *TMA* can reduce significantly the delivery cost of *Flood* for multicast data delivery.

Let us compare the delivery ratios among these three schemes. Fig. 8(c) shows the delivery ratio for the vehicle number. As discussed for the delivery delay, the higher vehicular density leads to the shorter delivery delay, so increases the delivery success probability for the limited packet lifetime (i.e., TTL). Thus, the packet can be delivered to the multicast group vehicles with a higher probability, indicating the high delivery ratio. *Flood* always has 100% delivery ratio regardless of vehicular density, but *TMA* and *Random* have higher delivery ratio as the vehicle number increases. From this figure, it can be seen that *TMA* has 99% delivery ratio except low

vehicular density (i.e., N=60 and N=80). As expected, Random has low delivery ratio (i.e., from 53% to 79%).

Therefore, through the optimal target point selection for the multicast group vehicles, *TMA* has better performance than *Random* and *Flood* in terms of packet delivery cost. This indicates the importance of an optimal target point selection for the multicast data delivery.

C. Impact of Vehicle Speed

In this subsection, we investigate how the change of mean vehicle speed affects the delivery cost, delivery delay, and delivery ratio. For the delivery cost, as shown in the Fig. 9(a), *TMA* outperforms *Random* and *Flood*. The higher vehicle speed leads to the lower delivery cost. For *TMA*, it can be seen that the vehicle speed up to 35MPH is helpful to reduce the overall multicast delivery cost, but the higher speed does not contribute much on the delivery cost reduction.

As shown in the Fig. 9(b), for TMA, Random and Flood, the higher vehicle speed leads to the shorter delivery delay. This is because the high vehicle speed yields high vehicle arrival rate at each road segment, leading to the shorter delivery delay. Note that at low speeds (i.e., 20 and 25MPH), Random has shorter delivery delay than TMA. This is because TMA's optimization is focused on the delivery cost rather than the delivery delay. For the delivery ratio, as shown in Fig. 9(c),

the TMA has much better performance than Random.

D. Impact of Vehicle Speed Deviation

In this subsection, we investigate the impact of vehicle speed deviation on the performance. We found that under a variety of vehicle speed deviations, *TMA* provides a lower delivery cost, a shorter delivery delay, and a higher delivery ratio than *Random*. Also, for the range of vehicle speed deviation, all of three schemes have almost constant curves for three performance parameters. For the packet delivery cost, as shown in Fig. 10(a), *TMA*, *Random* and *Flood* have the stable curves over the range of the vehicle speed deviation.

Fig. 10(b) shows the delivery delay according to the vehicle speed deviation. The delay performance gaps among these three schemes are almost constant at all of the vehicle speed deviations from 1 MPH to 10 MPH. However, for the delivery ratio, as shown in Fig. 10(c), *TMA* provides a reliable delivery close to 99%, however *Random* has the worse performance, about 73% delivery ratio.

Therefore, through the performance evaluation, we can conclude that *TMA* is a promising approach for the reliable, efficient multicast data delivery in vehicular networks through the *Multi-Anycast* based on the trajectories of multicast group vehicles.

VII. RELATED WORK

Recently, the VANET research has put a lot of attention on the data forwarding for vehicle-to-vehicle or vehicle-to-infrastructure communications [1]–[6]. Most of them are focused on the unicast data forwarding in vehicular networks.

Many data forwarding schemes (e.g., VADD [5], Delay Bounded Routing [3], and SADV [20]) are investigating the layout of road network and vehicular traffic statistics. VADD investigates the data forwarding based on a stochastic model to achieve the *lowest delivery delay* from vehicle to AP. On the other hand, Delay Bounded Routing proposes data forwarding schemes to satisfy the user-defined delay bound rather than the lowest delivery delay, while minimizing the channel utilization. SADV [20] first proposes a forwarding structure leveraging relay nodes for reliable data delivery. TBD [27] utilizes vehicle trajectory information along with vehicular traffic statistics for shorter delivery delay and better delivery probability for vehicle-to-infrastructure data delivery. TSF [19] first provides the forwarding for multi-hop infrastructure-tovehicle data delivery, based on vehicle trajectory. For all those existing approaches, they focus on the unicast data forwarding. On the other hand, TMA investigates the infrastructure-tovehicle multicast data delivery, utilizing the trajectories of those multicast group vehicles.

For the multicast in vehicular networks, Sebastian et al. [9] propose an efficient multicast dissemination scheme for the driving safety. The proposed scheme constructs a microscoped multicast tree consisting of vehicles moving on the same road segment. On the other hand, our *TMA* constructs a macro-scoped multicast tree consisting of relay nodes at intersections in the target road network. Kihl et al. [10]

propose a reliable geographical multicast routing in vehicular ad-hoc networks. This multicast routing forms a multicast tree by using a reactive route discovery for the multicast group vehicles. This approach is not viable for a large-scale road network due to the overhead of the control messages for the route discovery for the multicast group vehicles. On the other hand, *TMA* takes advantage of the trajectories of the multicast group vehicles to identify their locations in road networks without any control message.

VIII. CONCLUSION

In this paper, we propose <u>Trajectory-based Multi-Anycast</u> (TMA) for multicast data delivery in vehicular networks. Our goal is to provide a reliable, efficient multicast data delivery by minimizing the packet delivery cost (i.e., channel utilization) at the required data delivery ratio. This goal is achieved by computing packet-and-vehicle-rendezvous-points (called target points) for the data delivery to multicast group vehicles with the vehicle delay distribution and the packet delay distribution. These distributions can be obtained from the vehicle trajectory and the vehicular traffic statistics. Once optimal target points are determined for the multicast group vehicles, our TMA algorithm constructs a Delivery-Ratio Constrained Minimum Steiner Tree from the AP to the mobile multicast group vehicles. Data packets with the multicast tree encoding are source-routed from AP to the packet destinations along the multicast tree. With GPS-based navigation systems and DSRC communication devices, our TMA shows the effectiveness of vehicle trajectory in the multicast data delivery for the efficient data sharing in vehicular networks. As future work, we will explore the deployment issue of infrastructure nodes to support Quality-of-Service in large-scale road networks.

REFERENCES

- J. Eriksson, H. Balakrishnan, and S. Madden, "Cabernet: Vehicular Content Delivery Using WiFi," in MOBICOM. ACM, Sep. 2008.
- [2] N. Banerjee, M. D. Corner, D. Towsley, and B. N. Levine, "Relays, Base Stations, and Meshes: Enhancing Mobile Networks with Infrastructure," in MOBICOM. ACM, Sep. 2008.
- [3] A. Skordylis and N. Trigoni, "Delay-bounded Routing in Vehicular Adhoc Networks," in MOBIHOC. ACM, May 2008.
- [4] V. Naumov and T. R. Gross, "Connectivity-Aware Routing (CAR) in Vehicular Ad Hoc Networks," in *INFOCOM*. IEEE, May 2007.
- [5] J. Zhao and G. Cao, "VADD: Vehicle-Assisted Data Delivery in Vehicular Ad Hoc Networks," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 3, pp. 1910–1922, May 2008.
- [6] J. Ott and D. Kutscher, "Drive-thru Internet: IEEE 802.11b For "Automobile" Users," in INFOCOM. IEEE, Mar. 2004.
- [7] A. Carter, "The Status of Vehicle-to-Vehicle Communication as a Means of Improving Crash Prevention Performance," Tech. Rep. 05-0264, 2005, http://www-nrd.nhtsa.dot.gov/pdf/nrd-01/esv/esv19/05-0264-W.pdf.
- [8] H. Yomogita, "Mobile GPS Accelerates Chip Development," http://techon.nikkeibp.co.jp/article/HONSHI/20070424/131605/.
- [9] A. Sebastian, M. Tang, Y. Feng, and M. Looi, "A Multicast Routing Scheme for Efficient Safety Message Dissemination in VANET," in WCNC. IEEE, Apr. 2010.
- [10] M. Kihl, M. Sichitiu, T. Ekeroth, and M. Rozenberg, "Reliable Geographical Multicast Routing in Vehicular Ad-Hoc Networks," *LNCS*, vol. 4517, pp. 315–325, 2007.
- [11] M. Parsa, Q. Zhu, and J. Garcia-Luna-Aceves, "An Iterative Algorithm for Delay-Constrained Minimum-Cost Multicasting," *IEEE Transactions* on *Networking*, vol. 6, no. 4, pp. 461–474, Aug. 1998.

- [12] V. Bychkovsky, B. Hull, A. Miu, H. Balakrishnan, and S. Madden, "A Measurement Study of Vehicular Internet Access Using In Situ Wi-Fi Networks," in MOBICOM. ACM, Sep. 2006.
- [13] Jupiter Research, "Municipal Wireless: Partner to Spread Risks and Costs While Maximizing Benefit Opportunities," Tech. Rep., Jun. 2005.
- [14] Philadelphia Department of Transportation, "Traffic Control Center," http://philadelphia.pahighways.com/philadelphiatcc.html.
- [15] General Motors (GM), "Vehicle-to-Vehicle (V2V) Communications," http://www.gm.com/experience/technology/research/overview/isl/ vcim.jsp.
- [16] Toyota Motor Corporation (TMC), "TMC Develops Onboard DSRC Unit to Improve Traffic Safety," http://www2.toyota.co.jp/en/news/09/ 09/0903 html
- [17] Garmin Ltd., "Garmin Traffic," http://www8.garmin.com/traffic/.
- [18] Savari Networks, "StreetWAVE: Roadside Unit," http://www.savarinetworks.com/files/StreetWAVE-DS-final.pdf.
- [19] J. Jeong, S. Guo, Y. Gu, T. He, and D. Du, "TSF: Trajectory-based Statistical Forwarding for Infrastructure-to-Vehicle Data Delivery in Vehicular Networks," in *ICDCS*. IEEE, Jun. 2010.
- [20] Y. Ding, C. Wang, and L. Xiao, "A Static-Node Assisted Adaptive Routing Protocol in Vehicular Networks," in VANET. ACM, Sep. 2007.
- [21] M. DeGroot and M. Schervish, Probability and Statistics (3rd Edition). Addison-Wesley, 2001.
- [22] A. Polus, "A Study of Travel Time and Reliability on Arterial Routes," *Transportation*, vol. 8, no. 2, pp. 141–151, Jun. 1979.
- [23] D. S. Berry and D. M. Belmont, "Distribution of Vehicle Speeds and Travel Times," in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, Jul. 1950.
- [24] M. MacDougall, Simulating Computer Systems: Techniques and Tools. MIT Press, 1987.
- [25] T. Camp, J. Boleng, and V. Davies, "A Survey of Mobility Models for Ad Hoc Network Research," Wireless Communications and Mobility Computing (WCMC): Special Issue on Mobile Ad Hoc Networking: Research, Trends and Applications, vol. 2, pp. 483–502, Aug. 2002.
- [26] F. Bai, N. Sadagopan, and A. Helmy, "IMPORTANT: A framework to systematically analyze the Impact of Mobility on Performance of RouTing protocols for Adhoc NeTworks," in *INFOCOM*. IEEE, Mar. 2003.
- [27] J. Jeong, S. Guo, Y. Gu, T. He, and D. Du, "TBD: Trajectory-Based Data Forwarding for Light-Traffic Vehicular Networks," in *ICDCS*. IEEE, Jun. 2009.

APPENDIX A AVERAGE FORWARDING DISTANCE

The average forwarding distance $E[l_f]$ can be computed as the expected sum of the inter-distances D_h for h=1..k within the network component, as shown in Fig. 2. Suppose that the inter-arrival time T_h is exponentially distributed with arrival rate λ . So T_h for h=1..k are i.i.d. for the exponential distribution with parameter λ . Note the relationship between the inter-distance D_h and the inter-arrival time T_h that $D_h=vT_h$. Let a=R/v; that is, a is the time taken for a vehicle to move out of the communication range R with speed v. Let C(k) be the condition for the connected vehicular ad-hoc network consisting of k vehicle inter-arrivals (as shown in Fig. 2) such that C(k): $T_0 > a$ and $T_h \le a$ for h = 1..k. Let L(k) be the length of the connected ad-hoc network consisting of k vehicle inter-arrivals. Then, $E[l_f]$ can be derived using

the law of total expectation as follows:

$$E[l_f] = E[L] = \sum_{k=1}^{\infty} E[L(k)|C(k)] \times P[C(k)]$$

$$= v \times \sum_{k=1}^{\infty} E[\sum_{h=1}^{k} T_h|T_0 > a, T_h \le a \text{ for } h = 1..k]$$

$$\times P[T_0 > a, T_h \le a \text{ for } h = 1..k]$$

$$= E[vT_h|vT_h \le R] \times \frac{P[vT_h \le R]}{P[vT_h > R]}$$

$$= E[D_h|D_h \le R] \times \frac{P[D_h \le R]}{P[D_h > R]}.$$
(8)

APPENDIX B MEAN LINK DELAY

The mean link delay for road segment (I_i, I_j) of length l is computed considering the two cases in Fig. 2: (i) Immediate Forward and (ii) Wait and Carry. Suppose that the vehicles arrive with arrival rate λ . Let C(k) be the condition for the ad-hoc network consisting of k vehicle inter-arrivals. Let L(k) be the length of the connected ad-hoc network of k vehicle inter-arrivals. Therefore, the mean link delay E[d] is computed by the sum of the conditional expectations for the two cases in (1) as follows:

$$E[d] = \left(\sum_{k=1}^{\infty} E\left[\frac{l-R-L(k)}{v}|C(k)\right] \times P[C(k)]\right)$$

$$\times P[\text{forward}] + \left(E[\text{waiting time}] + \frac{l-R}{v}\right) \times P[\text{wait}]$$

$$= \frac{l-R-E[l_f]}{v}\beta + \left(\frac{1}{\lambda} + \frac{l-R}{v}\right)(1-\beta)$$
(9)

where $P[\text{forward}] = \beta = 1 - e^{-\frac{\lambda R}{v}}$, $P[\text{wait}] = 1 - \beta = e^{-\frac{\lambda R}{v}}$, and $E[\text{waiting time}] = \frac{1}{\lambda}$. Please, refer to Appendix A for the forwarding distance derivation related to L(k), C(k) and $E[l_f]$.

APPENDIX C VARIANCE OF LINK DELAY

For the variance of link delay, the second moment of link delay $E[d^2]$ is computed as follows:

$$E[d^{2}] = (\sum_{k=1}^{\infty} E[(\frac{l-R-L(k)}{v})^{2}|C(k)] \times P[C(k)])$$

$$\times P[\text{forward}] + (E[\text{waiting time}] + \frac{l-R}{v})^{2} \times P[\text{wait}]$$

$$= \frac{(l-R)^{2} - 2(l-R)E[l_{f}] + E[l_{f}^{2}]}{v^{2}}\beta$$

$$+ (\frac{1}{\lambda} + \frac{l-R}{v})^{2}(1-\beta).$$
(10)

Therefore, the link delay variance Var[d] is computed from (9) and (10) as follows: $Var[d] = E[d^2] - (E[d])^2$.