APPENDIX A
EMERGENCY MANEUVER PLANNING

In this section, we extend the vehicles' emergency maneuvers based on the criticality of the CNP risk assessment. The cluster head CH orchestrates the maneuvers of the cluster members CMs from the closest CM to the farthest CM from the obstacle. A goal position that an emergency vehicle $n_{em} \in \mathcal{V}_E$ drives toward is called a target position t_{pos}, in the maneuver lane. A maneuver reference path (p_{ref}) from the contour area of the vehicle n_{em} has t_{pos} as its ending point.

A. CNP Emergency Vehicle Maneuver

The minimal contour tracking defines both the control path and its tracking.

1) Minimal Contour Based Path Planning: During the maneuver, the first task is to define the maneuver input $u = \begin{bmatrix}\rho & a\end{bmatrix}^T$ that guarantees n_{em} to safely bypass the obstacle. This control input is the minimum steering angle that directs n_{em} toward t_{pos}. It is taken from n_{em}'s contour area defined by the steering angle input.

$$u = \min\left(\begin{bmatrix}\rho^1\\a^1\end{bmatrix}, \begin{bmatrix}\rho^2\\a^2\end{bmatrix}, ..., \begin{bmatrix}\rho^k\\a^k\end{bmatrix}\right), \tag{15}$$

where

ρ = steering angle input,
a = acceleration input,
$\begin{bmatrix}\rho^j\\a^j\end{bmatrix}, j \leq k$ = contour definition inputs.

Thus, we obtain an emergency control input from the input that results in an emergency vehicle's minimal contour area.

Definition A.1 (Minimal Contour Polygon Area of a Vehicle)**.** *Let the* ***Minimal Contour Polygon Area of a Vehicle*** *be a range formed by a series of n positions ζ that a vehicle can reach within the interval time Δ_t. For a steering angle $\rho \in [\rho_{min}, \rho_{max}]$, the paths of the vehicle are $\zeta_p = \{Path_{\rho min}...Path_{\rho max}\}$. Path-time step positions are $Path_\rho = \{P_{to}, P_{t1}, ...P_{tk}\}$ for $k = \frac{\Delta_t}{\delta}$.*

Given the current position P_t of a path, the next position $P_{t+\delta}$ is computed as follows:

$$\begin{aligned} P_{t+\delta} &= P_t + f(a_t, \rho_t) \\ &= (x_t, y_t) + v_t\delta(\cos(\theta_{t+\delta}), \sin(\theta_{t+\delta})), \end{aligned} \tag{16}$$

where $\theta_{t+\delta} = \theta_t + \frac{v_{t+\delta}}{L\tan(\rho_t)}$ and $v_{t+\delta} = v_t + a_t\delta$. Our algorithm decides $P_{ref} \in \zeta_p$ such that it heads to a destination that is as close to the t_{pos} as possible.

$$P_{ref} \in \zeta_p \text{ for} \arg\min[dist(t_{pos}, P_{tk})]. \tag{17}$$

A vehicle n_{em} avoids collisions with an obstacle n_{ob} or other vehicles throughout an interval time Δ_t by following a maneuver reference path P_{ref}. For each step time δ, the control input (ρ, a) that defines n_{em}'s next position will always direct to P_{t+1} as close to P_{ref} as possible. The maneuver process also ensures the non-degradation of the remaining vehicles' safety that might result from n_{em}'s movement. The emergency trajectories not only rely on the kinematics of the currently driving vehicles, but also flexibly adopt to the arrivals of new vehicles in the emergency road.

B. Sequential Probabilistic Control Definition

The probability of n_1^E colliding with n_1^L while maneuvering toward the left lane L_L to avoid a collision with n_{ob} is computed as:

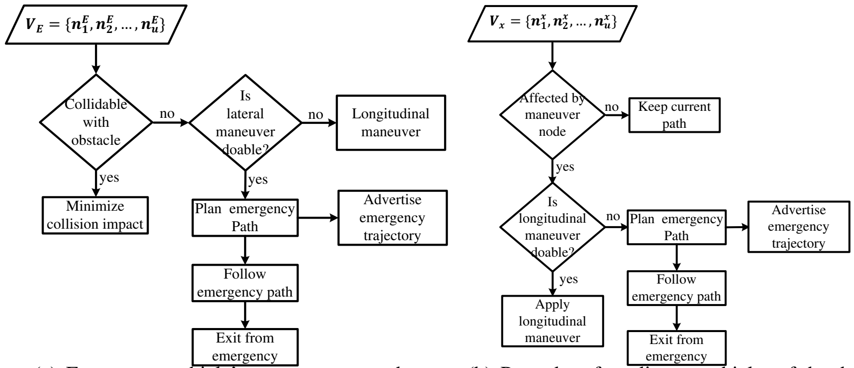
$$P(n_1^E \otimes n_1^L) = \frac{\mathcal{A}_1^L \cap \mathcal{A}_1^E}{\mathcal{A}_1^E}, \tag{18}$$

where $\mathcal{A}_1^L$ and $\mathcal{A}_1^E$ are the minimal contour polygons' areas respectively created by vehicles n_1^L and n_1^E during the interval time Δ_t. Fig. 13(a) illustrates the contours of three adjacent vehicles driving in a three lane-road which has an obstacle in the middle lane. Fig. 13(b) shows the contour intersection area that has been taken into account while calculating the appropriate n_1^E's maneuver to avoid future collision risks.

Algorithm 2 Compute Control Input

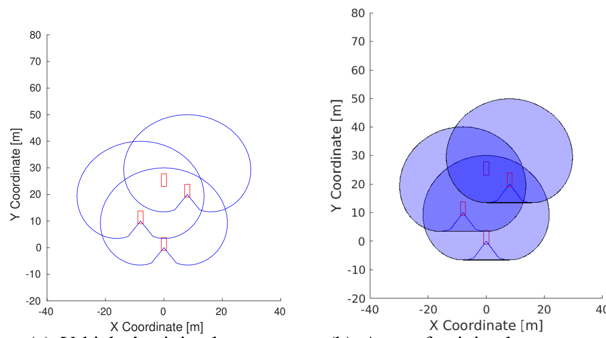
$LSD \leftarrow (x_0, y_0, v_0, \theta_0, \rho_0)$ ▷ Local sensor data (LSD) are data sensed by a vehicle's on-board unit (OBU): position (x_0, y_0), speed v_0, direction θ_0, and current steering angle ρ_0, respectively.
$RSD \leftarrow (x_i, y_i, v_i, \theta_i)$ ▷ Remote sensor data (RSD) are neighbors' data received through communication. Each neighbor i is represented by its position (x_i, y_i), speed v_i and direction θ_i.
function COMPUTE_EMERGENCY_CONTROL_INPUTS(n_1^E)
    $t_{pos} \leftarrow Define\_Target\_Position(x_{nob}, y_{nob})$ ▷ (x_{nob}, y_{nob}) is the obstacle's position from RSD.
    $\zeta_{n_1^E} \leftarrow Compute\_Contour\_Polygon\_Area()$
    $A_{inter} \leftarrow \emptyset$
    for each neighbor vehicle in L_x **do** ▷ L_x is the maneuver lane resulting from Algorithm 1.
        $\zeta_{n_1^L} \leftarrow Compute\_Contour\_Polygon\_Area()$
        $A_v \leftarrow \zeta_{n_1^E} \cap \zeta_{n_1^L}$
        if $A_v \neq \emptyset$ **then**
            $A_{inter} \leftarrow A_{inter} \cup A_v$
        end if
    end for
    $a \leftarrow 0$ ▷ Initialize the acceleration input a of n_{em}.
    $\rho \leftarrow 0$ ▷ Initialize the steering angle input ρ of n_{em}.
    if $A_{inter} = \emptyset$ **then** ▷ No risk of colliding with neighbors. The algorithm defines the steering angle and maintains its speed while changing lanes.
        $\rho \leftarrow Compute\_Best\_Steering\_Angle(t_{pos})$
    else ▷ Compute both the acceleration and steering needed to avoid the collision.
        $(\rho, a) \leftarrow Compute\_Maneuver\_Input(t_{pos}, A_{inter})$
    end if
    $u \leftarrow (\rho, a)$
    return u ▷ Return the maneuver input.
end function

We maneuver n_1^E within Δ_t in consideration of the neighbors' contour areas. The control input $\begin{bmatrix}\rho_i & a_i\end{bmatrix}^T$ shall guarantee the safe conditional collision probability toward the neighboring vehicles. This collision probability that n_1^E collides



(a) Emergency vehicle's maneuver procedure (b) Procedure for adjacent vehicles of the the emergency vehicle

Fig. 12. Cluster members' maneuver processing flows toward obstacle avoidance.



(a) Vehicles' minimal contours (b) Area of minimal contour polygons

Fig. 13. Contours of adjacent vehicles in an emergency situation.

with two vehicles in the maneuver lane will be calculated by the independent and identically distributed (IID) as follows:

$$\begin{aligned} P(n_2^L \otimes n_1^L \otimes n_1^E) &= P(n_2^L \otimes n_1^L | n_1^L \otimes n_1^E) \\ &= P(n_2^L \otimes n_1^L) P(n_1^L \otimes n_1^E). \end{aligned} \tag{19}$$

Considering a maneuver lane wherein three vehicles are currently driving, the collision probability for four vehicles is computed in the same way as follows:

$$\begin{aligned} P(n_3^L \otimes n_2^L \otimes n_1^L \otimes n_1^E) &= P(n_2^L \otimes n_3^L | n_2^L \otimes n_1^E) \\ &= P(n_3^L \otimes n_2^L) P(n_2^L \otimes n_1^L) P(n_1^L \otimes n_1^E). \end{aligned} \tag{20}$$

By generalizing the collision probability with the number of vehicles (u) driving in L, the collision probability becomes a chain of conditional probabilities as follows:

$$\begin{aligned} & P(n_u^L \otimes n_{u-1}^L \otimes ... \otimes n_1^L \otimes n_1^E) \\ &= P(n_u^L \otimes n_{u-1}^L | n_{u-1}^L \otimes n_{u-2}^L) \\ &= P(n_1^L \otimes n_1^E) \prod_{i=u}^{2} P(n_i^L \otimes n_{i-1}^L). \end{aligned} \tag{21}$$

C. Control Input Computational Algorithm

To define n_1^E's maneuver input, the CNP considers the risks of colliding with the maneuvering vehicles in a chosen lane. Algorithm 2 shows the process of computing the control input (ρ_t, a_t) toward the target position t_{pos}. Lines 1-2 provide sensed data as inputs of the algorithm. Line 4 selects the target position in the maneuver lane defined by Algorithm 1. Line 5 calculates the emergency vehicle's contour area. The next lines examine the proper control inputs of n_1^E while taking the other driving vehicles into consideration.

The contour-based risk analysis is performed in terms of an intersection area according to (18). Line 6 initializes the intersection area of the neighbor vehicles' contour areas. In lines 7-13, the comparison of n_1^E's contour polygon area with those of the neighboring vehicles is made to examine the collision risks during any maneuvers. When the contours have no intersection, a vehicle's maneuver will follow the definition of the steering angle ρ_t resulting from (17). Lines 14-15 initialize the control input. Lines 16-21 define the control input according to the neighboring vehicles' risks. After defining the needed maneuver input $u = (\rho, a)$, it is returned in line 22.

APPENDIX B
COLLISION STRENGTH MINIMIZATION

If the collision probability of an emergency vehicle n_{em} is obtained from (8a), then the vehicle is in inevitable collision state (ICS) and it will definitely crash. For this unavoidable situation, a collision strength minimization mechanism is needed to minimize the energy transfer between the colliding vehicles and to limit the number of vehicles involved in the collision to the possible extent.

The severity of a collision is proportional to the masses of two colliding vehicles and their corresponding speeds.

Assuming that an obstacle n_{ob} with speed v_{ob} and mass m_{ob} collides with n_{em} with speed v_e and mass m_e, the collision strength calculation is made using their Equivalent Energy Speed (EES). Knowing the approximate resultant speed v_r after a collision, the EES can be calculated as:

$$EES = v_r - v_{ob} = \frac{2m_e}{m_e + m_{ob}}(v_e - v_r). \tag{22}$$

The EES computation for vehicles driving in different directions, such as collisions from changing the current lane or overstepping the front vehicle, will result in a more general calculation form that involves driving direction θ_e. The speed that n_{em} applies to n_{ob} during a collision is:

$$\acute{v_e} = v_e \cos(\theta_e), \tag{23}$$

then, the EES will be computed as

$$EES = v_r - v_{ob} = \frac{2m_e}{m_e + m_{ob}}(\acute{v_e} - v_r). \tag{24}$$

The greater the difference in vehicle speeds is during a collision, the more severe the collision is. Autonomous emergency braking, when the vehicle is in the ICS, reduces the collision severity. Decelerating n_{em} just before colliding with n_{ob} can reduce the energy transfer between them. It is clear that if the speed difference is closer to null during a collision, EES will only depend on mass difference. The deceleration of the vehicle just before the collision is:

$$Dec = \frac{v_r - v_e}{\Delta_t}. \tag{25}$$

Thus, if $v_r = 0$ km/h during a collision the EES is nullified. This means that the damages from a collision are negligible. The lower the speed is, the lower EES is. Another aspect of minimizing the collision strength is to reduce the number of involved vehicles. CNP has communication-based situation-awareness to notify the driving vehicle of a possible collision in advance. Prior decisions are required to steer the next vehicle before arriving at the point of collision. This reduces the number of vehicles involved in an accident and avoids a chain of collisions.